MODEL QUESTION FOR END SEMESTER.

M.Sc. Sem- IV Paper - ECMATH403(C) 45 (1)

(Integral Equations)

Short type questions

1) If $K(x,t)$ is symmetric and $f_0(x)$ and $f_1(x)$ are eigen functions of $K(x,t)$ corresponding to eigen values λ_0 and λ_1 respectively $[\lambda_0 \neq \lambda_1]$ Then, Prove that $f_0(x)$ and $f_1(x)$ are orthogonal on $[a,b]$.

2) Prove that the eigen values of a symmetric kernel are real.

3) Show that $y(x) = \cos 2x$ is a solution of the integral equation

$$y(x) = \cos x + 3\int_0^{\pi} K(x,t)\, y(t)\, dt.$$

where,

$$K(x,t) = \begin{cases} \sin x \cdot \cos t & ; 0 \le x \le t \\ \cos x \cdot \sin t & ; t \le x \le \pi \end{cases}$$

4) Find the eigen functions of the Homogeneous Integral Equation

$$u(x) = \lambda \int_{-1}^{1} \{5xt^3 + 4x^2 t + 3tx\}\, u(t)\, dt.$$

5) Define a) An Integral Equation
b) Symmetric Kernel
c) Seperable Kernel
d) Iterated Kernel
e) General form of Abel Singular Integral equation.

6) Find the Resolvent Kernel of the Volterra Integral Equation with Kernel

a) $K(x,t) = e^{x-t}$

b) $K(x,t) = 1$

7) Form an integral equation corresponding to the differential eq^n given by

$$y'' - 2xy = 0$$

with initial Conditions

$$y(0) = \frac{1}{2}, \quad y'(0) = 1, \quad y''(0) = 1$$

Long-type questions:

1) Let $u(x) = f(x) + \lambda \int_a^b K(x,t)\, u(t)\, dt$ —— (1)

be a Fredholm Integral Equation of Second Kind such that

i) Kernel $K(x,t) \neq 0$ is real and Continuous in a rectangle R $(a \le x \le b,\ a \le t \le b)$
Also,
$K(x,t)$ is bounded by M in R
i.e. $|K(x,t)| \le M$ in R.

ii) $f(x) \neq 0$ is real and continuous in the interval I, for which $a \le x \le b$
Also, let $|f(x)| \le N$ in I.

iii) λ is a constant such that

$$|\lambda| < \frac{1}{M(b-a)}$$

Then Prove that eq^n ① has a unique continuous solution in I which is given by

$$u(x) = f(x) + \lambda \int_a^b k(x,t) f(t)\, dt + \lambda^2 \int_a^b k(x,t) \int_a^b k(t,t_1)$$

$$f(t_1)\, dt_1\, dt + \cdots\cdots$$

$\boxed{\text{or}}$

Find the solution of Fredholm Integral Equation of the second kind by Successive substitution.

2) Use the method of Successive approximation solve the integral equation

$$u(x) = 1 + \int_0^x (x-t)\, u(t)\, dt \quad , \quad u_0(x) = 0$$

3) Solve FIE of second kind with seperable Kernels.

4) Solve the Integral Equation

$$u(x) = \cos x + \lambda \int_0^\pi \sin(x-t)\, u(t)\, dt.$$

5) Reduce the Boundary Value Problem into an integral equation

$$u''(x) + \lambda u = 0$$

with Boundary Conditions

$$u(0) = 0 \quad , \quad u(l) = 0$$

6) Solve

$$u(x) = x + \lambda \int_0^1 (1+x+t)\, u(t)\, dt$$

7) Solve homogeneous FIE of the second Kind

$$u(x) = \lambda \int_0^{2\pi} \sin(x+t)\, u(t)\, dt.$$

8) By means of Resolvent Kernel, find the solution of Integral Equation

a) $u(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\, u(t)\, dt.$

b) $u(x) = x + \int_0^x (t-x)\, u(t)\, dt$

9) Find the solution of Abel's Integral Eq^n.

10) Solve

$$f(x) = \int_x^b \frac{y(t)\, dt}{(\cos x - \cos t)^{1/2}} \; ; \; 0 \leq a \leq x \leq b \leq \pi$$